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STRUCTURE OF COMPRESSION SHOCK WAVES IN POROUS ELASTOPLASTIC MATERIALS

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The shock-wave structure in a porous elastoplastic material is studied. In a certain range of parameters, the existence of a four-wave structure of a compression shock wave is possible. Regimes in which a reflected shock wave does not appear at all have been found in the problem of shock-wave reflection from a rigid wall. In this case, the entire energy of the incident shock wave transforms to thermal energy due to dissipation induced by the viscous collapse of the pores.

For certainty, we consider the propagation of a shock wave (SW) in a porous material in a one-dimensional case for which Kiselev and Fomin [1] proposed equations of continuity, motion, energy, and state. In the present paper, we use only those equations which are necessary for a qualitative analysis of the SW structure. Directing the x_1 axis of the Cartesian coordinate system perpendicular to the SW front, we determine the main values of the strain rate tensor

$$\dot{\varepsilon}_1 = \frac{\partial v}{\partial x}, \qquad \dot{\varepsilon}_2 = \dot{\varepsilon}_3 = 0 \tag{1}$$

and the main values of the stress tensor

$$\sigma_1 = S_1 - p, \qquad S_2 = S_3, \qquad S_1 + S_2 + S_3 = 0, \tag{2}$$

where v is the material velocity along the x_1 axis, $\dot{\varepsilon}_i$, σ_i, and S_i are the main values of the strain rate, stress, and stress deviator tensors. The pressure p is determined from the equation

$$p = K\Big(\frac{\rho_s}{\rho_s^0} - 1\Big), \qquad \rho = \rho_s m_2, \qquad m_1 + m_2 = 1. \tag{3}$$

Here ρ_s is the density of the material, ρ is the mean density of the porous medium, m_1 is the porosity, m_2 is the volume concentration of the material, which determines the fraction of the unit volume occupied by the material, and K is the volume compression modulus. The superscript 0 here and in what follows corresponds to the initial unstressed state. In formula (3), we ignore the thermal pressure, which corresponds formally to $T = 0$ and, according to the Nernst theorem, $S = 0$, where T is the temperature and S is the entropy. This approximation is valid for weak shock waves and low porosity. In this case, these conditions are assumed to be satisfied. In the numerical calculations presented below, we consider weak shock waves in steel with a pressure $p < 10$ GPa behind the shock wave and a low porosity $m_1 < 10^{-2}$. We note that the influence of thermal effects is taken into account in the numerical solution, since a complete system of equations from [1] is solved; however, as the calculation shows, the contribution of the thermal pressure to the total pressure is very small. Hence, Eqs. (2) and (3) define the adiabat of the porous material $\sigma_1 = \sigma_1(\rho)$. The function $S_1(\rho)$ is found from the following conditions of uniaxial deformation: the deformation of the porous material is elastic for

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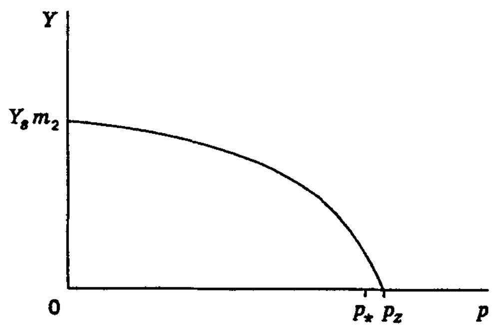

Fig. 1

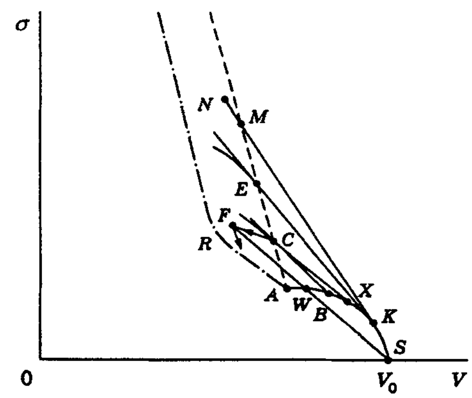

Fig. 2

$|S_1| < 2Y/3$ and elastoplastic for $|S_1| = 2Y/3$. According to [1], we present the increment of S_1 in the form

$$dS_1 = \begin{cases} -4\mu\, d\rho/(3\rho), & |S_1| < 2Y/3, \\ -2dY/3, & |S_1| = 2Y/3, \end{cases} \tag{4}$$

where μ is the shear module and Y is the yield strength of a porous material, which depends on the pressure p and porosity m_1 [1]. A formula that defines $Y = Y(p, m_1)$ is given in [1]. In what follows, we need only the qualitative dependence $Y(p)$ for $m_1 = \text{const}$ shown in Fig. 1. The pressure p_z determines the point where the yield strength vanishes; therefore, $Y = 0$ for $p \geqslant p_z$. The quantity p_* determines the pressure at which the collapse of the pores begins. It is found from the formula

$$p_* = (2/3)\, Y_s \ln(1/\bar{m}_1), \qquad \bar{m}_1 = k m_1, \qquad k = 1.7. \tag{5}$$

For the porosity $m_1 < 10^{-2}$ the pressures p_z and p_* are close to each other [1], and in the qualitative analysis we assume that $p_* \approx p_z$. The change in porosity caused by the collapse of the pores is described by the equation

$$\frac{\dot{m}_1}{m_1} = \frac{3}{4}\frac{p_* - p}{\eta}, \qquad p > p_*. \tag{6}$$

Using (1)–(6), we plot the qualitative dependence $\sigma = \sigma(V)$, where $\sigma = -\sigma_1$ and $V = 1/\rho$ is the specific volume, which is called the adiabat of the porous material (Fig. 2). On the section SK we have an elastic load $|S_1| < 2Y/3$, and from (1)–(4) we obtain

$$\sigma = p - S_1, \qquad S_1 = -(4/3)\mu(V_0/V - 1). \tag{7}$$

The pressure $p = p(V)$ is found from (3), where we should assume that $\rho = 1/V$. The yield condition $|S_1| = 2Y/3$ is satisfied at the point K; therefore, on the section KA we have

$$\sigma = p + \frac{2}{3} Y(p), \tag{8}$$

where $p = p(V)$ is determined from (3) and the point A is found from the condition $Y = 0$. If the stress is $\sigma > \sigma_A$ (σ_A is the pressure at the point A), the pores in the porous body collapse. In this case, for a given V, the pressure lies within the interval between the equilibrium adiabat $p_r(V)$ (the dot-and-dashed curve AR) and the frozen adiabat $p_f(V)$ (the dashed curve AM). The equilibrium adiabat $p_r(V)$ describes slow processes where each value of pressure corresponds to an equilibrium value of porosity. Assuming $\dot{m}_1 = 0$ in Eq. (6) and taking into account Eq. (3), we obtain

$$p_r = \frac{2}{3} Y_s \ln\left(\frac{1}{\bar{m}_1}\right), \qquad \frac{2}{3}\frac{Y_s}{K} \ln\left(\frac{1}{\bar{m}_1}\right) + \frac{V_0 m_2^0}{V(1 - m_1)} - 1 = 0. \tag{9}$$

Slow processes are also those in which the characteristic time of variation of the mean parameters Δt

is significantly greater than the relaxation time τ related to the collapse of the pores. From Eq. (6) it follows that the characteristic relaxation time can be evaluated by the formula $\tau \approx \eta/(p_* - p)$. The reverse inequality $\Delta t \ll \tau$ is valid for fast processes. The frozen adiabat $p_f(V)$ describes fast processes in which the porosity does not have enough time to change significantly, and the frozen adiabat equation follows from formula (3) if we assume that $m_2 = m_2^0$:

$$p_f = p_A + KV_0\left(\frac{1}{V} - \frac{1}{V_A}\right). \tag{10}$$

Knowing the adiabat $\sigma(V)$ (Fig. 2), we can determine the propagation velocity of small perturbations (the speed of sound)

$$c = \sqrt{\left(\frac{\partial \sigma}{\partial \rho}\right)_s} = \sqrt{-V^2 \frac{\partial \sigma}{\partial V}}. \tag{11}$$

Using formulas (7), (8), and (11), we find the speed of sound in the elastic region SK

$$c_e = \sqrt{\left(K + \frac{4}{3}\mu\right)/\rho_0},$$

and in the plastic region it is

$$c_p = \sqrt{\left(1 + \frac{2}{3}\frac{\partial Y}{\partial \rho}\right)\frac{\partial p}{\partial \rho}}. \tag{12}$$

It follows from [1] that the derivative $\partial Y/\partial p \to -3/2$ as $p \to p_z$; therefore, from (12) we obtain $c_p \to 0$ as $p \to p_z$. In the region $p > p_z$ there is a spectrum of the speed of sound from equilibrium c_r to frozen c_f, which are determined from (9)–(11):

$$0 < c_r < c < c_f, \qquad c_f = \sqrt{\frac{K}{\rho}}, \qquad c_r = \sqrt{\frac{K}{m_2 \rho_s^0}\left(1 - \left(1 + \frac{2Y_s}{3K_s}\frac{m_2^2 V}{m_1 m_2^0 V_0}\right)^{-1}\right)}.$$

The resultant relationships for the adiabat and the speeds of sound allow us to analyze qualitatively the propagation of a shock wave in a porous material.

Let a shock wave propagate in a porous material. The final state behind this SW is located at the point F that lies on the intersection of the adiabat and the Rayleigh line $\sigma_F = j^2(V_0 - V_F)$ that issues out of the point S ($j = v/V$ is the mass flux). Since the adiabat SKA is convex, the Rayleigh line crosses it at a certain point W, which leads to an ambiguous solution. Hence, such an SW cannot exist. It splits into several waves that follow one another. Since the steepest section is SK, the first wave is an elastic precursor with velocity c_e and stress σ_K. Then follows a plastic wave whose velocity c_p decreases as σ increases, which leads to the smearing of the wave in time. At the point A the speed of sound increases instantly from c_p to c_r. The intersection of characteristics results in the emergence of a new shock wave. This new SW, in turn, splits into a frozen wave BC and a relaxation wave CF (Figs. 2 and 3). The SW speed relative to the matter ahead of the front $D = V_B\sqrt{(\sigma_C - \sigma_B)/(V_B - V_C)}$ is greater than the speed of sound c_p at the point B; therefore, the point B shifts down on the adiabat to the point X where the SW velocity equals the speed of sound $[D = c_p(X)]$.

At the point C the speed of sound is $c < D$ (see Fig. 2), and the frozen wave BC propagates faster than the relaxation wave CF. As noted above, there is a whole spectrum of speeds of sound from $p > p_*$ to c_r in a porous material for c_f. It follows from Fig. 2 that $c_f > D$; hence, the condition of evolution of the frozen SW relative to the frozen speed of sound is valid. A region of uniform flow cannot exist between the relaxation and frozen shock waves. From the equations of motion and continuity [1] and the relation $\dot{m}_2 > 0$ it follows that the following inequalities are valid for $\sigma > \sigma_C$:

$$\dot{\rho} > 0, \qquad \frac{\partial v}{\partial x} < 0, \qquad \frac{\partial \sigma}{\partial x} < 0.$$

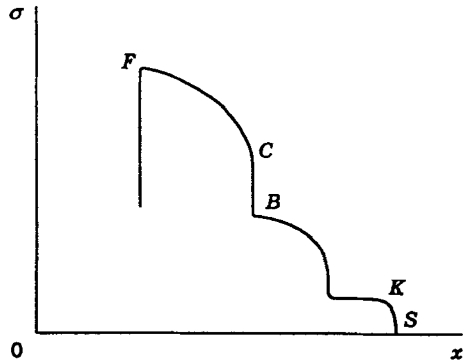

Fig. 3

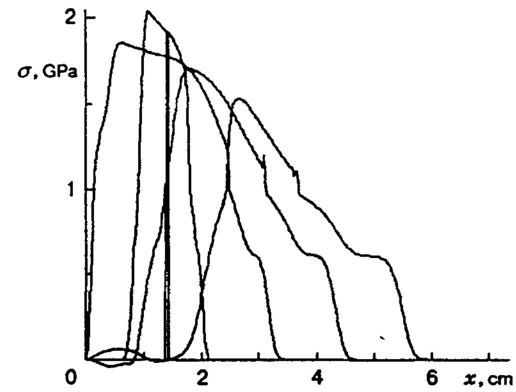

Fig. 4

Hence, the relaxation SW is a compression wave FC (see Fig. 3). The amplitude of the relaxation SW decreases with time, since the rarefaction SW approaches and weakens it. Figure 4 shows wave profiles $\sigma(x)$ at several times from 1 to 7 μsec with a step $\Delta t = 2$ μsec that were obtained as a result of the numerical solution of the complete system of equations of [1] in the problem of collision of plates. A continuous steel plate of thickness $h_1 = 1.4$ cm hit a porous steel plate of thickness $h_2 = 5.8$ cm and porosity $m_2 = 0.02$ with a velocity $v_p = 0.120$ mm/μsec. The strength parameters of steel were chosen equal to those in [1]: $\rho_s = 7.85$ g/cm^3, $K = 160$ GPa, $\mu = 80$ GPa, $Y_s \approx 0.4$ GPa, and viscosity of steel $\eta = 3 \cdot 10^3$ Pa·sec. The vertical solid curve in Fig. 4 corresponds to the contact boundary between the porous and continuous materials. It is seen that the SW has a four-wave structure and the amplitude of the relaxation SW decreases with time. As long as the rarefaction wave did not enter the porous material, the maximum stress in the relaxation wave remained constant and the width increased. In the porous material the rarefaction wave transforms to a shock rarefaction wave which decreases the amplitude and the width of the relaxation SW.

An increase in the impact velocity v_p and, hence, the SW velocity leads to an increase in the angle of inclination of the Rayleigh line; therefore, the velocity of the frozen SW at some point E (see Fig. 2) equals the relaxation wave velocity. As a result, the amplitude of the frozen SW is determined from the Chapman–Jouguet-type condition at the points E and X. It follows that the amplitude of the frozen SW remains almost constant. For a greater SW velocity the final state is found at the point N, the amplitude of the frozen SW being determined from the intersection of the Rayleigh line and the frozen adiabat (the point M in Fig. 2). An increase in the SW velocity shifts the point X toward the point K. For a certain SW velocity these points coincide, and the shock wave consists of an elastic precursor and a plastic shock wave following behind it (a two-wave configuration).

Of interest are the features that arise upon the SW reflection from a rigid wall. If the shock wave amplitude is rather high and a complete collapse of the pores occurs behind the shock wave, then, owing to the nonlinearity of $\sigma(V)$, the SW reflection factor $k = \sigma_r/\sigma_i$ is greater than two (σ_i is the stress behind the incident SW and σ_r is the stress behind the reflected SW). To prove it, we note that if the stress difference in the SW is $\Delta\sigma \leqslant 0.3$ Mbar, the velocity difference Δu is found from the formula $\Delta u = \Delta\sigma/\rho c$. Hence, the amplitudes of the incident and reflected waves are $\sigma_i = z_i u_i$ and $\sigma_r = z_r u_r$, where $z_i = \rho_i c_i$ and $z_r = \rho_r c_r$, the subscripts i and r referring to the incident SW and reflected SW, respectively. From the rigid wall condition we obtain $u_i = u_r = v_p$, and for the reflection factor we have $k = 1 + z_r/z_i$. A complete collapse of the pores occurs behind the incident SW, and the reflected wave propagates in a continuous material whose acoustic impedance is $z_s = \rho_s c_s$. By virtue of the inequalities $c_s > c_f > c_i$ and $\rho_s > \rho_i$ and the condition $z_r = z_s$ we find that the reflection factor in this case satisfies the inequality $k > 2$. A decrease in the amplitude of the incident SW leads to the fact that the collapse of the pores behind it is incomplete. As a result, the reflected SW propagates in a porous material, the difference between the values of z_r and z_i decreases, and, consequently, the reflection factor k decreases. Finally, another case is possible where the pores do not collapse behind the

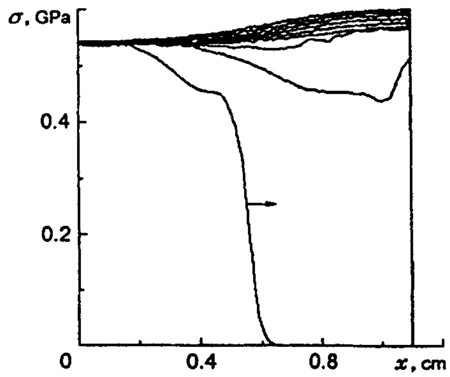

Fig. 5

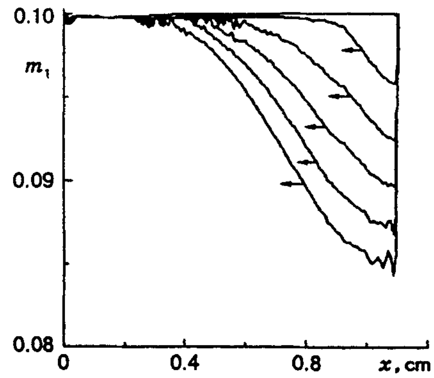

Fig. 6

incident SW, but the entire process of collapse occurs behind the reflected SW; then we have $z_i \approx z_s$ and $z_r \ll z_i$. In this case, the reflected SW does not appear at all ($k \approx 1$). The energy transferred by the incident SW completely transforms to thermal energy due to the viscous collapse of the pores. This situation is similar to an absolutely inelastic impact where the entire kinetic energy of the colliding bodies transforms to heat.

Figure 5 shows distributions of stresses $\sigma(x)$ that arise in porous steel at times $t = 1, 2, \ldots, 8$ μsec counted from the moment when the incident SW hits the rigid wall. The incident SW was induced by a piston that moved with a constant velocity $v_p = 0.016$ mm/μsec in a material with porosity $m_1^0 = 0.1$. The rigid wall bounded the material on the right at $x = 1.1$ cm. It is seen that the amplitude of the reflected SW tends to zero and its width tends to infinity. The width of the relaxation SW is determined by the characteristic time of the collapse of the pores [$\Delta x \sim c_r\tau$ and $\tau \sim \eta/(p - p_*)$], and for $p \sim p_*$ we have $\Delta x \to \infty$. Figure 6 shows the porosity m_1 versus the coordinate x that was calculated for the same times. It is seen that in this case the collapse of the pores begins in the vicinity of the rigid wall behind the reflected SW. As the reflected SW propagates, the region of collapse of the pores extends and the porosity decreases. A further decrease in the amplitude of the reflected SW leads to the appearance of a reflected elastoplastic wave with an amplitude $p_* - \sigma_i$, behind which a relaxation wave of small amplitude and large width propagates. The amplitude of the elastoplastic wave increases as v_p decreases. At a certain value of v_p the pores do not collapse behind the SW reflected from the rigid wall either. In this case, we have $z_r \approx z_i \approx z_s$, and the reflection factor increases to $k \approx 2$.

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